Unsteady Flow (transient)

When fluid properties change with respect to time then the flow is unsteady.

Examples:
- Filling the tank
- Emptying (discharging) the tank.

Let $m_i$ and $m_e$ be the masses entering and leaving the control volume.
- Let $m_1$ & $m_2$ be masses in the control volume at the initial & final conditions.

1. Control volume condition initially
2. Control volume condition finally
i. Inlet condition
e. Exit condition

![Diagram with masses](image)

\[ m_i = 6 \text{ kg} \]
\[ m_1 = 10 \text{ kg} \]
\[ m_2 = 14 \text{ kg} \]
\[ m_e = 2 \text{ kg} \]

\[ m_1 = 6 \]
\[ m_2 = 23 \]
\[ m_e = 3 \]

\[ m_{i1} = 20 \]
\[ m_{e1} = 3 \]
\[ m_{i2} = 23 \]
\[ m_{e2} = 3 \]

\[ (\frac{dm}{dt})_{cv} = \frac{dm_i}{dt} - \frac{dm_e}{dt} \]

\[ m = \text{kg} \]
\[ m = \text{kg/s} \]

\[ (\frac{dm}{dt})_{cv} = \dot{m}_i - \dot{m}_e \]

$\dot{m}_i =$ mass entering per unit time
$\dot{m}_e =$ mass leaving per unit time.
II. conservation of Energy -

\[
\left( \frac{dE}{dt} \right)_{cv} = \frac{dE_i}{dt} - \frac{dE_e}{dt}
\]

\[ h_i + \frac{c_i^2}{2} + gzi + Q - h_e + \frac{c_e^2}{2} + gze + \omega \]

Energy at inlet

\[ E_i = m_i h_i + \frac{1}{2} m_i c_i^2 + m_i gzi + \Omega \]

\[ E_e = m_e h_e + \frac{1}{2} m_e c_e^2 + m_e gze + W \]

\[ \left( \frac{dE}{dt} \right)_{cv} = \frac{d}{dt} \left[ m_i h_i + \frac{1}{2} m_i c_i^2 + m_i gzi + \Omega \right] - \frac{d}{dt} \left[ m_e h_e + \frac{1}{2} m_e c_e^2 + m_e gze + W \right] \]

\[ E = KE + PE + U \]

\[ dE = d(KE) + d(PE) + dU \]

\[ dE = dU \quad \text{if kinetic and potential energy changes are neglected} \]

Assumption 1: Neglect KE & PE changes

\[ \left( \frac{du}{dt} \right)_{cv} = \frac{d}{dt} \left[ m_i h_i + \Omega \right] - \frac{d}{dt} \left[ m_e h_e + W \right] \]

Assumption 2: If \( h_i \) and \( h_e \) do not change with respect to time.

\[ \left( \frac{du}{dt} \right)_{cv} = h_i \frac{dm_i}{dt} + \dot{\Omega} - h_e \frac{dme}{dt} - \dot{W} \]

\[ \left( \frac{du}{dt} \right)_{cv} = h_i m_i + \dot{\Omega} - h_e \dot{me} - \dot{W} \quad \text{(2)} \]
Conventional Questions:

Q.1. An insulated storage tank is initially evacuated and is connected to a supply pipe line carrying a fluid at a specific internal energy $u_i$ and specific enthalpy $h_i$; the valve is opened and the fluid flows into the tank from supply pipe line and reaches the pressure same as that of supply pipe line. Show that the final specific internal energy of the fluid in a tank is equal to $h_i$, and hence deduce if the fluid is an ideal gas the final temp, $T_2$ of the gas in the tank is $\sqrt{Y}$ times supply line temperature.

Sol.

\[
\left( \frac{dm}{dt} \right)_{cv} = \dot{m}_i - \dot{m}_e
\]

As no mass is leaving in $cv$ i.e. \( \dot{m}_e = 0 \)

\[
\left( \frac{dm}{dt} \right)_{cv} = \dot{m}_i \quad \text{---- (1)}
\]

\[
\left( \frac{du}{dt} \right)_{c.v.} = h_i \dot{m}_i + \dot{Q} - h_e \dot{m}_e - \dot{W}
\]

\[
\left( \frac{du}{dt} \right)_{c.v.} = h_i \dot{m}_i \quad \text{----- (2)}
\]

Substitute (1) in (2)

\[
\left( \frac{du}{dt} \right)_{c.v.} = h_i \left( \frac{dm}{dt} \right)_{c.v}
\]

Integrating

\[
\left( \frac{du}{dt} \right)_{c.v.} = h_i (dm)_{cv}
\]

\[
(U_2 - U_1) = h_i (m_2 - m_1)
\]

\[
m_2 U_2 - m_1 U_1 = h_i (m_2 - m_1)
\]

\[
\begin{align*}
\{ & U_1 = 0 \\
& m_1 = 0 \\
& m_2 U_2 = h_i m_2^2 \\
\end{align*}
\]
If the fluid is an ideal gas

$$c_v T_2 = c_p T_1$$

$$T_2 = \frac{c_p}{c_v} T_1$$

$$T_2 = Y T_i$$

Q.2 An insulated pressure cylinder of volume $V$ contains air at a pressure $P_i$ and temperature $T_i$; it is filled by supply air pipeline maintained at a pressure $P_i$ and temperature $T_i$. Show that the final temperature of air $T_2$ in the cylinder after it has been charged to a pressure $P_i$ is given by

$$T_2 = \frac{Y T_i}{1 + \frac{P_i}{P_i} \left( \frac{Y T_i}{T_i} - 1 \right)}$$

$$\left( \frac{dm}{dt} \right)_{c.v} = m_i - m_e$$

$$\left( \frac{dm}{dt} \right)_{c.v} = m_i \quad (1)$$

$$\left( \frac{du}{dt} \right)_{c.v} = h_i m_i + \frac{Q}{\dot{m}_i} - h_2 m_e - \dot{W}$$

$$\left( \frac{du}{dt} \right)_{c.v} = h_i \left( \frac{dm}{dt} \right)_{c.v}$$

$$\left( du \right)_{c.v} = h_i \left( dm \right)_{c.v}$$
\[(U_2 - U_1) = h_i (m_2 - m_1)\]
\[\Rightarrow m_2 u_2 - m_1 u_1 = h_i (m_2 - m_1)\]
\[\Rightarrow m_2 \cdot \frac{\gamma \nu T_2}{\gamma \nu T_1} - m_1 \frac{\gamma \nu T_1}{\gamma \nu T_1} = \frac{c_p}{c_v} T_i \left( m_2 - m_1 \right)\]
\[\Rightarrow \frac{P_i V}{RT_2} \cdot T_2 - \frac{P_i V}{RT_1} \times T_1 = \gamma T_i \left( \frac{P_i V}{RT_2} - \frac{P_i V}{RT_1} \right)\]
\[\Rightarrow P_i V - P_i V = \gamma T_i \left[ \frac{P_i V}{T_2} - \frac{P_i V}{T_1} \right]\]
\[\Rightarrow \frac{P_i V - P_i V}{\frac{P_i V T_1 - P_i V T_2}{T_1 T_1}} = \gamma T_i\]
\[\Rightarrow \frac{P_i V - P_i V}{P_i V T_1 - P_i V T_2} = \gamma T_i\]
\[\Rightarrow P_i - P_i = \gamma T_i \left[ \frac{P_i V}{T_2} - \frac{P_i V}{T_1} \right]\]
\[\Rightarrow P_i - P_i = \gamma T_i \times \frac{P_i}{T_2} - \gamma T_i \frac{P_i}{T_1}\]
\[\Rightarrow \gamma T_i \times \frac{P_i}{T_2} = P_i - P_i + \gamma T_i \frac{P_i}{T_1}\]
\[\Rightarrow \gamma T_i \times \frac{P_i}{T_2} = P_i \left[ 1 + \frac{P_i}{P_i} \left( \gamma T_i \frac{P_i}{T_1} - 1 \right) \right]\]

\[T_2 = \frac{\gamma T_i}{\left( 1 + \frac{P_i}{P_i} \left[ \frac{\gamma T_i}{T_1} - 1 \right] \right)}\]