Mobility Analysis


Mechanism (Means of achieving Something)

→ motion transformation

→ Eq: IC engine

piston translation is converted to crank rotation

Sewing Machine - rotation motion of $\ell$/$\rho$ is converted to reciprocating motion of needle.

Elements of a Mechanism:

1. Kinematic link: part/element of a system which is connecting at least two other entities and allow relative motion b/w them.

Singlar link:

Binary link: There are two connecting points on the link.

Ternary link: (3 connecting points)

or

Quaternary link:

or many

⇒ If two links of a mechanism do not have any relative motion they both/ all are to be considered as one link.
**kinematic pair**: connection between 2 kinematic link that allows certain type of relative motion to be present between the link.

**Classification of kinematic pair**:

(a) Based on no. of links connected at a pairing location.

1. Binary kinematic pair

![Diagram of binary kinematic pair](image)

\[ \text{No. of Binary kinematic pair} = \text{No. of link} - 1 \]
\[ \Rightarrow 3 - 1 = 2 \]

2. Ternary kinematic pair:

![Diagram of ternary kinematic pair](image)

3. Quaternary kinematic pair:

![Diagram of quaternary kinematic pair](image)

\[ \text{No. of Binary kinematic pair} = \text{No. of link} - 1 \]
\[ \Rightarrow 4 - 1 = 3 \]

\[ \text{No. of Binary kinematic pair} = n - 1 \]

\( n \) = no. of link connected at pairing location

(b) Based on nature of relative motion between links.

1. **Revolute pair** (R): links have relative rotation about pairing location.

2. **Rotary/Turning pair** (another name): every point of one link has circular motion with pairing location as centre wrt other link.

\[ \theta \rightarrow \text{pair variable} \]
(2) Prismatic pair (p):

Prismatic: Member having same cross-section shape & size along its longitudinal direction.

Links have relative translation. Every point of one link wrt other link has parallel straight line path.

\[ S = \text{pair variable.} \]

(3) Helical pair (H)/ Screw pair:
Ex: Bolt & Nut

Dependent/constrained \( R + T \)

\[ L/P = \text{lead/pitch of helix} \]

\[ S = \frac{\theta}{2\pi} L \]

One link wrt other link has relative motion i.e. every point of one link wrt other link has coaxial helical path of same helix angle.

(4) Cylindrical pair:-(c)

Independent \( R + T \)

\[ \theta = \frac{S}{L} \]
links have relative cylindrical (i.e. independent \( R+T \) motion).

Every point on one link has cylindrical path coaxial with axis of cylindrical pair \( \text{wrt} \) other link.

(5) **Spherical pair(s):** links have relative spherical motion.

![Fig: Ball-socket joint](image)

Every point on one link has concentric spherical path \( \text{wrt} \) other link.

(6) **Planar pair (E):**

\( E - \text{Ebene (plane)} \)

links have relative planer motion. Every point on one link \( \text{wrt} \) other link move in parallel plane.

\[ \begin{align*}
\theta_x (\alpha) \\
\theta_y (\beta) \\
\theta_z (\gamma)
\end{align*} \]

\((\theta_x, \theta_y, \theta_z)\)

(7) **Rolling pair**

when Relative motion is pure rolling

\[ V_{CM} = R \omega \]

\[ \begin{align*}
R \omega & \quad 1 \\
& \quad V_{CM}
\end{align*} \]

\( \Rightarrow 2 \text{ independent translation} + 1 \text{ indep rotation.} \)
(c) Based on no. of independent relative motion:

No. of unconstrained unconnected independent relative motions = 6
Degree of freedom = 6

Connected freedom: 1-5 (in contact)

One body has ≤ 3 ind. relative motion wrt another body, the pairing is termed as lower.

⇒ Single point contact → No. of degree of freedom = 5
Two point contact b/w the bodies

\[ T_x \vee T_y \times T_z \vee \]
\[ R_x \vee R_y \vee R_z \times \]
D.O.F = 4

When 2 point contact is mathematically same as line contact.
Three point contact:

Degree of Freedom = 3

Three point in contact mathematically same as area contact.

Lower pairs → kp that allows ≤ 3 degree of freedom b/w links
  Ex: R, P, H, C, S, E ⇒ surfaces in contact are same.
  → physically realised by surface/area contact.

Higher pair → kp that allows > 3 (i.e. 4/5) DOP b/w the links
  physically realised by line/point contact.

Wrapping pair:

(d) Based on closure:
  how the contact b/w links is maintained.

(i) Form closed -
  Geometry
  closed pair
(2) Force closed open pair

1

Gravity

2

Spring

1

(1) If contact between link is maintained geometrically of link then it is formed closed pair.

(2) If contact between links is maintained due to force of gravity/spring then it is forced closed pair.

Kinematic pair in 2D:-

only Revolute and prismatic are lower pairs in 2D.

unconnected body in a plane has 3 DOF

1 point contact → Removes 1T ; DOF = 2

2 point contact → Removes 2T ; DOF = 1

point contact in plane → HP (2 DOF)

line contact in plane → LP (1 DOF)

 LP → R, P.

(a) Open loop KC (Serial KC):

at least one link is connected to only one another link
App — Robotics, protein synthesis.

(b) Closed KC (Parallel):

Every link is connected to at least 2 other links.

Mechanism:— closed KC with one of its link fixed.

Linkage—Mechanism with only lower pair.

Mobility of Mechanism:

No. of independent inputs required to drive the mechanism so that every links have definite Relative motion also called as D.O.F. of Mechanism.

Kutzbach criterion for Mobility:—

$$\text{LP = 2J}_1$$

unconnected freedom of 'n' links = 3n

No. of LP/ KP with 1 DOF = J_1

No. of HP/ KP with 2 DOF = J_2

Degree of constrained imposed by LP = 2J_1
Degree of constrained imposed by \( HP = J_2 \)

mobility of kinematic chain = \( 3n - 2J_1 - J_2 \)

\( \Rightarrow \) If one of the link is fixed DOF imposed by fixity = 3

\( \Rightarrow \) Mobility of Mechanism = \( 3n - 2J_1 - J_2 - 3 \)

\[
\text{m/ D.O.F. } \Rightarrow 3(n-1) - 2J_1 - J_2
\]

3D:

\[
m = 6(n-1) - 5J_1 - 4J_2 - 3J_3 - 2J_4 - J_5
\]

\[
m = 6(n-1) - \sum_{i=1}^{5} (6-i) J_i
\]

2D:

\( m = 1 \) constrained Mechanism

for every unique position of input link, there is unique position for all other links (one-to-one mapping)

\( m = 0 \rightarrow \text{not a mechanism it is a structure} \)

\( m < 0 \rightarrow \text{Statically indeterminate structure.} \)

\( m > 1 \) unconstrained Mechanism

for unique position of i/p links there need not be unique position of other links.

(One-to-many mapping)

\( \Rightarrow \) Nature of Relative motion b/w 2 links:

(1) Completely constrained Motion (DOF = 1):

Only one relative motion (either rotation or translation is present)
(2) **Incompletely constrained motion**:

more than one relative motion is possible between two links.

(3) **Successfully constrained motion**:

only one relative motion is allowed by removing others by applying external force/ Torque.

(4) Syringe.

fig: foot step bearing.
piston-cylinder arrangement - cylindrical pair

Successfully constrained.

**Constrained Mechanism:** \((m=1)\)

\[
m = 3(n-1) - 2J_1 - J_2 = 1
\]

\[
\Rightarrow 3n - 3 - 2J_1 - J_2 = 1
\]

\[
n = \frac{2J_1 + J_2 + 4}{3}
\]

Min. No. of links in a planer Mechanism = 3, and it has 2 LP and 1 HP.

Constrained linkage:

\[
J_2 = 0
\]

\[
m = 1 = 3(n-1) - 2J
\]

\[
1 = 3n - 3 - 2J
\]

\[
n = \frac{2J + 4}{3}
\]

\[
| \begin{array}{ccc}
J_1 & J_2 \\
2 & 1 & 0 \\
x & 1 & 1 \\
x & 2 & 0 \\
\end{array} |
\]

Min. No. of link in a constrained planer linkage = 4

\[
| \begin{array}{ccc}
J & x \\
2 & 3 \\
\end{array} |
\]

Note: No. of links in a constrained planer linkage is always even.
No of ternary link in a constrained planer linkage for 

\[ n > 4 \]

\[ = 2(j - n) \]

for 6 bar linkage = 2(7-6) = 2 ternary link.

\[ n = \frac{2j + 4}{3} \]

\[ 3n - 2j - 4 = 0 \]

For \( m = 1 \).

\[ \text{link} = i + i - 1 + 1 = \frac{2i}{(n)} \]

\[ m = 3(n-1) - 2j \]

\[ j = i + 2(i - 2) + 1 + 1 \]

\[ \Rightarrow 3i - 2 \]

\[ m = 1 = 3(3i - 2) - 2j \quad 3(n-1) - 2j \]

\[ 3i - 2 = 9i - 1 \Rightarrow 3(n-1) - 2(3i - 2) \]
\[
1 = 3n - 3 - 6i + 4
\]
\[
3n = 6i
\]
\[
\therefore n = 2i
\]
\[
i = \frac{n}{2}
\]

For a constrained planar linkage, max. no. of connecting points on a single link = \(\frac{n}{2}\)

**Inversions of a Mechanism:**

(i)

![Diagram of mechanism](image)

1. Fixed \(S \downarrow \quad 2 \rightarrow \text{cw}(-\text{ve})

2. Fixed \(S \downarrow \quad 1 \rightarrow \text{ccw}(+\text{ve})

(ii)

![Diagram of mechanism](image)

Fixing 1 \(S \downarrow \quad \text{link } 2 \rightarrow -x

Fixing 2 \(S \downarrow \quad \text{link } 1 \rightarrow +x

(a)

![Diagram of mechanism](image)

\(S \downarrow \) for specified relative motion

\(\text{link } -2 \rightarrow \text{cw}(-\text{ve})

\(\text{link } 1 \rightarrow \text{ccw}(+\text{ve})

\text{abs. motion gets inverted.
For a given relative motion between links abs. motion of the 
links gets Inverted when the fixity of linked is 
changed. So, we get a variety of absolute motion of link 
by changing the fixity of link in a sequence. The process 
of obtaining different abs. motion by changing the fixity 
of links is called inversion.

**Inversion of 4-bar mechanism**

Grashof criteria for continuous rotatability of 
at least 1 link of the mechanism.

\[
m = \frac{2j+4}{3}
\]

\(s\) = length of shortest link

\(l\) = length of longest link

\(p, q\) = lengths of other 2 links.

**Grashof Four bar linkage**

\[s + l \leq p + q\]

Type-1 Grashof \(s + l < p + q\)

Type-2 Grashof \(s + l = p + q\)

Non-Grashof linkage \(s + l > p + q\)

**Inversion of Type-1 Grashof linkage**

(1) Link(s) adjacent to shortest link is fixed.
shortest link → crank
link opposite to shortest link → rocker

extreme position of rocker is obtained when crank & coupler are collinear/inline.

\[(s+l)^2 = p^2 + q^2 - 2pq \cos \beta_{\text{max}}\]

\[\beta_{\text{max}} = \cos^{-1}\left[\frac{p^2 + q^2 - (s+l)^2}{2pq}\right]\]

\[\beta_{\text{min}} = \cos^{-1}\left[\frac{p^2 + q^2 - (l-s)^2}{2pq}\right]\]

\[p^2 = (s+l)^2 + q^2 - 2(s+l)(q) \cos \theta_1\]

\[\theta_1 = \cos^{-1}\left[\frac{(s+l)^2 + q^2 - p^2}{2(s+l)q}\right]\]
\[ p^2 = (l-s)^2 + q^2 - 2(l-s)q \cos \theta_2 \]

\[ \theta_2 = \cos^{-1} \left[ \frac{(l-s)^2 + q^2 - p^2}{2(l-s)q} \right] \]

\[ t_f = \text{time of forward stroke} = \frac{\theta_f}{\omega} \]

\[ t_r = \text{time of return stroke} = \frac{\theta_r}{\omega} \]

Quick return ratio:

\[ \frac{t_f}{t_r} = \frac{\theta_f}{\theta_r} \]

\[ \theta_f + \theta_r = 360^\circ \]

\[ \phi = \theta_2 - \theta_1 \]

\[ \theta_f = 180 + \phi \]

\[ \theta_r = 180 - \phi \]

\[ \text{QRR} = \frac{t_f}{t_r} = \frac{\theta_f}{\theta_r} = \frac{180 + \phi}{180 - \phi} \]

\[ \text{QRR} = f(\phi) \]

Inversion III:

- Shortest link fixed - Double crank mechanism.
- Links adjacent to shortest link are crank.
- Link opposite to shortest link - coupler.
Inversion-III link opposite shortest link is fixed

⇒ Double rocker Mechanism

⇒ links adjacent to shortest link fixed links are rocker.

⇒ Shortest link can make continuous rotation.
Extreme position of output link (1) is obtained when input link and coupler is collinear:

\[ \Rightarrow \text{Extreme position of input link (q) is obtained when output link (1) and coupler (s) are collinear.} \]

**Dead lock condition**

For input link

For output link

Inversions of Type - II Grashof:

\[ s + l = p + q \]

\[ q \quad s = p, \quad l = q \]

(a) parallelogram linkage:

(Anti-parallelogram linkage):

\[ \Rightarrow \text{which ever link is fixed} \]

\[ \text{it is always a double crank mechanism.} \]

(Two equal length link are opposite to each other)
(b) **Deltoid linkage:**

Two equal length link adjacent to each other.

1. When shortest link is fixed, we get double crank mechanism.
2. When link other than shortest link (longest link) is fixed, it is called/we get double crank rocker mechanism.

**Inversion of non-Grashof:**

\[ s + l > r + q \]

Which ever link is fixed, it always results in a double rocker mechanism.

4-bar Mechanism \(\rightarrow\) 4R-linkage

**Single Slider Crank chain Mechanism**

3 Revolute/Turning pair + 1 Sliding/prismatic pair

**Slider-crank chain**

3R + 1P
Grashof criteria for Slider-crank chain

When $e=0 \rightarrow$ zero offset slider-crank chain.

No. of Inversions = No. of links = 4

Inversion-1: Link 1 is fixed $\rightarrow$ crank-rocker mechanism

Shortest link

Slider (Reciprocating Rocker)

Continuous Rotation is present

\( 2\theta = 0^\circ \) Slider will be at TDC/ODC, Dead Centre position $v_s = 0$

Extreme position of slider is obtained when crank/coupler are collinear.
\( \theta = 180^\circ \) Slider will be at BDC/IDC crank angle covered during slider motion from TDC to BDC and BDC to TDC is the same.

\( \theta_x = \theta_f = 180^\circ \) (equal return mechanism) \( e = 0 \)

\[ A_c_1 = \theta_x + l \]

\[ A_c_2 = l - \theta_x \]

\[ \theta_1 \theta_2 = \theta_f \text{ at centre of crank} \]

\[ \theta_2 \theta_1 = \theta_h \text{ at } -11 \]
\[ \theta_f + \theta_r = 360^\circ \]
\[ \theta_f = 180 + \alpha \]
\[ \theta_r = 180 - \alpha \]

\[ \cos \theta_2 = \frac{e}{l - e} \]
\[ \cos \theta_1 = \frac{e}{l + e} \]

\[ \theta_1 = \cos^{-1} \left( \frac{e}{l + e} \right) \]
\[ \theta_2 = \cos^{-1} \left( \frac{e}{l - e} \right) \]

\[ \alpha = \theta_1 - \theta_2 \rightarrow \text{From } \alpha \text{ we find } \theta_f, \theta_r \rightarrow \text{then} \]
\[ \text{We find QRR = } \frac{\theta_f}{\theta_r} \]

\[ \text{QRR = } \frac{t_f}{t_r} = \frac{\theta_f}{\theta_r} = \frac{180 + \alpha}{180 - \alpha} \]

When offset is downwards, suction stroke is more than compression stroke.

When offset is upwards suction stroke < compression stroke.
Length of stroke of piston:
\[ = c_1c_2 \]
\[ \Rightarrow DC_1 - DC_2 \]
\[ \Rightarrow \sqrt{(l + x_1)^2 - e^2} - \sqrt{(l - x_2)^2 - e^2} \]

\[ = \text{when } e = 0 \]

Stroke length = \( 2r \)

**Application:**

1. **power generating**
   - I.C. Engine
   - Slider - link 4 \( \rightarrow \) I/P
   - Crank - link 2 \( \rightarrow \) O/P
   - Link 3 \( \rightarrow \) Coupler - connecting rod.

**Inversion-II** \( \rightarrow \) link - 2 is fixed (Double - crank Mechanism)

![](image)

(2) **power consuming**
- Reciprocating pump/ compressor
  - Crank - link - 2 \( \rightarrow \) Input
  - Slider - link - 4 \( \rightarrow \) Output
  - Link 3 - Coupler.

link adjacent to shortest link are cranks

\[ \downarrow \text{link 1 & link 3.} \]

**Application**

1. **power generating**
   - Rotating cylinder engine
   - Link - 1 \( \rightarrow \) Rotating Cylinder - Coupler
   - Link - 4 \( \rightarrow \) Piston \( \rightarrow \) I/P
   - Link - 3 \( \rightarrow \) Crank \( \rightarrow \) O/P.
Fig: Rotating Cylinder engine.

(2) Power consuming: 
Whitworth Quick return Mechanism.

link3 - Crank - input
link1 - Crank - output
link 4 - Coupler

To attach the tool, we used first inversion with zero offset and crank of the tool connected to link 1 rigidly.

Extreme position of the tool is when crank of the tool is collinear with its connecting rod (link 5) i.e. when link 1 becomes ⊥ to fixed link 2.

\[ \theta_f > \theta_{\theta 2} \]

\[ \omega t_f > \omega t_{\theta 2} \]

\[ t_f > t_{\theta 2} \]
\[ \Theta_F = 2\alpha \]

\[ \cos \alpha = \frac{r_2}{l} \]

\[ \alpha = \cos^{-1} \left( \frac{r_2}{l} \right) \]

\[ \Theta_F = 360^\circ - \Theta_h = 360^\circ - 2\alpha \]

\[ \text{QRR} = \frac{t_F}{t_h} = \frac{\Theta_F}{\Theta_h} = \frac{360^\circ - \Theta_h}{\Theta_h} = \frac{360^\circ - 2\alpha}{2\alpha} \quad \text{Imp.} \]

\* \text{When} \quad \frac{r_2}{l} = \frac{1}{2} \quad (i.e. \ l = 2r_2) \quad \alpha = 60^\circ

\[ \text{QRR} = \frac{240}{120} = 2 \]
Inversion-3. Link-3 is fixed.

Link adjacent to shortest link is fixed, crank-rocker mechanism.

Link-2 → Crank
Link-1 (rocker) → Reciprocating wrt 4, oscillating wrt 4.

Application:

1. Power generating
   Oscillating cylinder engine
   Link-1 → oscillating cylinder-coupler
   Link-4 → piston I/P
   Link-2 → Crank → O/P.

2. Power consuming:
   Crank and slotted lever Quick return mechanism.

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\[ \theta_r = 2\alpha \]
\[ \theta_f = 360 - \theta_r \Rightarrow 360 - 2\alpha \]

\( w = \text{constant (assume)} \)

\[ QRR = \frac{\theta_f}{\theta_r} = \frac{t_f}{t_r} \]

\[ \Rightarrow \frac{360 - 2\alpha}{2\alpha} \]

\[ \cos \alpha = \frac{r_2}{l} ; \quad \text{if} \quad \frac{r_2}{l} = \frac{1}{2} \{ \text{or} \quad l = 2\alpha \} \]

\[ \alpha = 60^\circ \]

\[ QRR = 2. \]

**Inversion 4:** link 4 is fixed.

(If link opposite to shortest link is fixed → Double rocker)

Application:-

1. Hand pump mechanism

![Diagram of hand pump mechanism]
Double slider crank chain Mechanism:

2 Turning pair / Revolute pair + 2 prismatic pair / sliding pair

Inversion 1. Link 1 is fixed

Link 2/4 - input
Link 4/2 - output
Link 3 coupler

\[ x = BP \cos \theta \]
\[ y = AP \sin \theta \]

\[ \left( \frac{x}{BP} \right)^2 + \left( \frac{y}{AP} \right)^2 = 1 \rightarrow \text{equation of ellipse} \]
\[ \begin{align*}
A_P & \rightarrow \text{major radius} \\
B_P & \rightarrow \text{minor radius} \\
\Rightarrow \text{Elliptical trammel Mechanism.}^* \\
\text{When } A_P = B_P \text{ i.e. when } P \text{ is mid point of coupler, the } P \\
\text{traces a circle of radius } A_P = B_P. \\
\text{Inversion-2 Link 2/4 is fixed. (Skotch- Yoke Mechanism)}
\end{align*} \]

\[ \begin{align*}
\text{Link 3- } & \text{ I/P} \\
\text{Link 1- } & \text{01P} \\
\text{Link 4- } & \text{coupler.}
\end{align*} \]

\[
\begin{align*}
x &= r \cos \theta \\
\theta &= f(t) \\
\theta &= \omega t \\
\frac{d\theta}{dt} &= \omega \\
\text{velocity at point } P \quad \left( V_P \right) &= \frac{dx}{dt} = -r \sin \theta \cdot \omega \\
A_P &= -r \omega^2 \cos \theta \\
A_P &= -\omega^2 x
\end{align*}
\]

- \( P \) executes SHM.
- \( P \) is a point on link 1
- Link 1 executes SHM (Translation)
- \( \text{[Skotch- Yoke Mechanism]} \)
Inversion-3. Link 3 is fixed.

link - 2/4 - input  
link - 4/2 - output  
link-1 - coupler.

* Application

Oldham coupling  
(used for connecting two parallel shaft)  
are coupled in such a way that if one shaft rotates other shaft is rotated at the same speed).

⇒ link 1 simultaneously translates wrt 2 and 4  
max. velocity of rubbing of link 1 wrt 2/4 = ew.

Transmission angle:

\[
M_B = \overrightarrow{r}_{A/B} \times \overrightarrow{F} = |\overrightarrow{r}_{A/B}| |\overrightarrow{F}| \sin \theta
\]

\[\theta = 90^\circ \text{ (turning tendency)}\]
output torque is maximum.
Angle between coupler and output link is called transmission angle. Ideal value of $\mu = 90^\circ$.

$\mu = f(\theta)$

Generally $60^\circ < \mu < 120^\circ$

$BD^2 = s^2 + q^2 - 2sq\cos\theta$

$BD^2 = l^2 + p^2 - 2lp \cos \mu$

$\cos \mu = \frac{l^2 + p^2 - s^2 - q^2 + 2sq\cos\theta}{2lp}$

$\Rightarrow -\sin \mu \, d\mu = -\frac{sq \sin \theta \, d\theta}{lp}$

$\Rightarrow \frac{d\mu}{d\theta} = 0$ for finding at what value of $\theta$, $\mu$ is max/min.

$\Rightarrow \frac{d\mu}{d\theta} = \frac{sq \sin \theta}{lp \sin \mu} = 0 \Rightarrow \sin \theta = 0$

$\Rightarrow 0^\circ, 180^\circ$

$\mu_{\min}$

$\mu_{\max}$

$\theta = 0^\circ, 180^\circ$
Transmission angle for double rocker:

Theoretical it is possible for \( \mu \) to be zero, when input link is rotated further to the condition where input link is collinear with coupler. In such condition mechanism is moving towards dead/lock configuration where coupler and output link are collinear (\( \mu = 0 \)).

To avoid rotation of input link beyond certain position, mechanical stoppers may be used.

Practically transmission angle is taken to be \( \mu_{\text{min}} \) when input link is collinear with coupler and i.e. when o/p link is at extreme position.

\[
\rho^2 = (s+q)^2 + l^2 - 2l(s+q)\cos \mu_{\text{min}}
\]

\[
\cos \mu_{\text{min}} = \frac{(s+q)^2 + l^2 - \rho^2}{2(s+q)l}
\]

Transmission angle is max.
when input link is at extreme position i.e. when coupler link and o/p link collinear

\[ \mu_{\text{max}} = 180^\circ \]
In some cases in a double rocker mechanism, it is possible to have \( \theta = 0^\circ \) and \( \theta = 180^\circ \) in such cases \( \mu_{\text{max}} \) and \( \mu_{\text{min}} \) is obtained in a similar way as that of crank-rocker mechanism i.e. use the formula.

To see whether \( \theta = 0^\circ / \theta = 180^\circ \) is possible or not, substitute line length in the formula of \( \cos \mu \). If \( \cos \mu_{\text{max}} / \cos \mu_{\text{min}} \) is between \([-1,1]\) then \( \theta = 0^\circ \) and \( \theta = 180^\circ \) possible.

But if \( \cos \mu_{\text{max}} / \cos \mu_{\text{min}} \) are not between \([-1,1]\) (Eg: \( \cos \mu_{\text{max}} = 1 \)) that it can be inferred that \( \theta = 180^\circ / \theta = 0^\circ \) are not possible.

**Mechanical Advantage:**

\[
\text{MA} = \frac{\text{output effort}}{\text{Input Effort}} = \frac{F_2}{F_1} = \frac{l_1}{l_2}
\]

\( \text{if } l_1 > l_2 ; \text{ MA} > 1 \).

**MA OF 4 Revolute Crank Rocker Mechanism:**

All pairs are frictionless.

Due to no power loss.

\[
T_2 \omega_2 = T_4 \omega_4
\]

\[\text{MA} = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4}\]

*When rocker is at extreme p limit/toggle position \( \omega_4 \)

\[\text{MA} = \infty\]
Even for a small input torque output torque will be very large.

FIG: stone crusher Mechanism

Mechanical Advantage of slider crank mechanism:

(a) Reciprocating pump / compressor:

link 2 → input (crank)
link 4 → piston → output

\[ T_2 \omega_2 = F_4 v_4 \]

\[ = r_2 F_2 \omega_2 \]

\[ M_A = \frac{F_4}{F_2} = \frac{r_2 \omega_2}{v_4} \]

\[ v_4 = 0 \text{ at } TDC/BDC \]

\[ M_A \rightarrow \infty \]

(b) IC engine → piston I/P crank o/p

\[ M_A = \frac{F_2}{F_4} = \frac{v_4}{\lambda \omega_2} \text{ at } TDC/BDC \]

\[ M_A = 0 \]
Example: Mobility analysis:

(1) \[ n = 3, \quad J_1 = 2, \quad J_2 = 1 \]
\[ m = 3(n-1) - 2J_1 - J_2 \]
\[ 3(3-1) - 2 \times 2 - 1 \]
\[ 6 - 4 - 1 \]
\[ m = 1 \]

(2) \[ n = 3, \quad J_1 = 3, \quad J_2 = 0 \]
\[ m = 3(n-1) - 2J_1 - J_2 \]
\[ 3(3-1) - 2 \times 3 \]
\[ m = 0 \]

(3) \[ n = 3, \quad J_1 = 3, \quad J_2 = 0 \]
\[ m = 3(n-1) - 2J_1 - J_2 \]
\[ 3 \times 2 - 2 \times 3 \]
\[ m = 0 \]

Just criteria is failed here it is not a structure, it is a mechanism.

** m actual = 1

Limitation of Kutzbach criterion.

* criterion does not consider geometry of links (size and shape)

In this case freedom of 3 wrt 2 can be obtained can be even through a point contact having a line contact is redundant.
Criterion does not consider the nature of KP based on rel. motion in links, but considers KP based on no. of independent relative motion (DOF) of KP.

\[ m = 3(n-1) - 2j \]
\[ \Rightarrow 3(3-1) - 2 \times 3 \]
\[ m = 0 \]

Minimum no. of links in a constrained linkage = 3.

\[ \text{Single link} \]

\[ v_E = x_2 \omega_2 = v_B \]
\[ v_E = x_5 \omega_5 \]
\[ \omega_5 = 2 \omega_2 \]

Double parallelogram linkage

\[ n = 5, \quad J_i = 6 \]
\[ m = 3(n-1) - 2j - J_2 \]
\[ \Rightarrow 12 - 12 \]
\[ m = 0 \]

But \( M_{\text{actual}} = 1 \)

* Kuzmbach criterion does not consider the geometric configuration of mechanism.

- In this case even if link 5/4 is removed, then link 4/5 motion remains the same.

One of the links (5/4) is redundant kinematically.

\[ n_r = \text{No. of redundant link.} = 1 \]
\[ J_{r_k} = \text{No. of redundant KP} = 2 \]

\[ M_{\text{eff/act.}} = 3(n-1) - 2(J_i - J_r) - J_2 \]
\[ \Rightarrow 3(5-1-1) - 2(6-2) - 0 \Rightarrow 9 - 8 = 1. \text{ Ans.} \]
Double \( \|/ \) gram linkage is an exception to Kutzboch criterion.

\[
m = 3(n-1) - 2J_1 - J_2
\]
\[
= 3(5-1) - 2 \times 6
\]
\[
= 0 \quad \checkmark
\]

ABEF is a parallelogram linkage.

\[
F_r = \text{redundant freedom of a}
\]
\[
KP = 1
\]

\[
m_{act/eff} = 3(n-n_{fr}) - 2(J_1 - J_r)
\]
\[
- J_2 - J_r
\]
\[
\Rightarrow 3(4-1) - 2 \times 4 - 0 - 1
\]
\[
\Rightarrow 0
\]

* If a single link is paired with other links by more than 1 prismatic pair and all of them allow translatory of link in same dirn then the other \( p \) pairs are redundant.

\[
m = 3(4-1) - 2 \times 4 - 0
\]
\[
m = 1 \quad \checkmark
\]
Double \( m \) gram linkage is an exception to kutzbock criterion.

\[ m = 3(n-1) - 2J_1 - J_2 \]
\[ = 3(5-1) - 2 \times 6 \]
\[ m = 0 \]

\( \text{ABEF is a parallelogram linkage.} \)

\[ \text{Fr} = \text{redundent freedom of a} \]
\[ KP = 1 \]
\[ \text{mact/eff} = 3(n-n_f-1) - 2(J_1-J_f) - J_2 - \text{Fr} \]
\[ = 3(4-1) - 2 \times 4 - 0 - 1 \]
\[ = 0 \]

* If a single link is paired with other links by more than 1 prismatic pair and all of them allow translatory of link in same dirn then the other \( p \) pairs are redundant.
(g) \[ m = 3(u-1) - 2 \times 3 - 1 \]
\[ \Rightarrow 9 - 0 - 6 - 1 \]
\[ m = 2 \]
Fr = 1 = rev. pair freedom of 3 w.r.t.
Mact = m - Fr = 2 - 1 = 1
\( \rightarrow \) is for motion transformation of 2 w.r.t 4.

(10). \[ m = 3(u-1) - 2 \times 3 - 1 \]
\[ \Rightarrow 9 - 0 - 6 - 1 \]
\[ m = 2 \]

* (a) When disc roll w/o slipping

\[ V_c = r \omega \]
\[ S = r \theta \text{ eqn of constraint} \]

(Doc) = 1 additional due to condition of no slip.

Mact = m - 1 = 2 - 1 = \( \frac{1}{2} \) V

(b) When disc is rolling with slipping.

There are 2 independent motion for disc i.e. disc can translate and rotate independently.

[m=2]
(11)

\[ m = 3(n-1) - 2J_1 - J_2 \]
\[ n = 4 \quad J_1 = 3 \quad J_2 = 1 \]
\[ m = 9 - 6 - 1 \]
\[ m = 2 \]

(a) disc are not slipping wrt each other

(b) when disc roll with slipping:

Condition of no slip

\[ R\alpha = r\beta \]

Diff.

\[ R.\omega_2 = r\omega_3 \]

1 degree of constr

\[ m = 2 \]

\[ m = 2 + 1 = 3 \]

\[ \uparrow \text{is for slipping d.o.f.} \]

If discs are replaced are replaced by gears ther because no slipping exists between gears.